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Revisiting Zakat with a Distribution of Weighted Shapley Value

Purpose: This paper extends the Shapley Value (SV) into a discussion of Zakat, a Pillar of Islam. Lloyd Shapley was awarded the Nobel Prize in Economics in 2012. We show that their relationship is significant for all nations, that of levelling up. An important but neglected paper by Datta (1939) showed insights provided by the Power Law, or as it is sometimes called, the Pareto Distribution, into the role of Zakat in raising the income of all above the subsistence level. The Pareto Distribution describes the prevailing tendency.

The SV illustrates the interdependence perspective of Zakat with the Pareto distribution, wealth, income, and poverty. Payoffs apply equally to both givers and receivers. For our purposes, payoffs are considered as transferable utilities. They are formed by individuals who willingly cooperate in society rather than atomistic individuals who act independently. Zakat represents the recognition that society needs to be cooperative rather than individualistic; people cooperate in groups or societies to create value. Shapley's value implications and axioms are evaluated with an illustration.

Design/methodology/approach: We extend Datta's approach by introducing distribution weights into the SV. We set out the concept of weighted shapely values (SWV) that retain the elements of randomness and marginal contribution to a coalition contained in pure/true shapely values and weights that follow a ley-Pareto distribution. The paper is a viewpoint work that relies primarily on the author's qualitative interpretation.

Findings: The findings indicate that individual members of a coalition make multiple contributions that are often unrewarded. The contribution of one member of a coalition is dependent upon the contribution of others. The measure of contributions is payoffs, which have both monetary and non-monetary aspects; transferable payoffs or utilities are usually assumed. Furthermore, the significant agents in society or an organisation are stakeholders rather than the usual categories: managers, staff, shareholders, Etc.

Practical and theoretical implications: Contextualising these concepts within the Islamic values and principles that guide Zakat administration is crucial to ensure that the distribution of Zakat funds is fair, equitable, and meets the needs of all eligible recipients. By applying these concepts appropriately, Zakat administrators can ensure that the Zakat system functions effectively and fulfils its religious obligation.

Originality/value: The novelty of this paper is that it blends the SV and the idea behind Zakat by introducing the idea of alternatives of shapely weights. The link between the institution of Zakat and SV in terms of equality, poverty elimination and wealth distribution should be at the top of the research agenda.

Keywords: Weighted Shapley Value, Zakat/Tithe, Pareto/Levy Distribution, Zakat Administration, Shapley Value, Income and Wealth Distribution.

1. Introduction

This paper blends the shapely value and the idea behind Zakat by introducing the idea of alternatives in the form of shapely weights. Datta (1939) analysed the link between Zakat, a pillar of Islam to the practice of tithe in Medieval Christian society. He linked Zakat to the Pareto-Levy distribution a broad class that includes power-law distributions. They are a good approximation of income and wealth distribution today. Datta examines Zakat's economic and social impact on individuals, communities, and society as a whole. He discusses the historical and contemporary practices of Zakat and its role in poverty alleviation, wealth redistribution, and economic development.

Datta's paper recalls the historical tie between economics and religion, the perennial issue of levelling up, the fact that no *one* is an island, independent of others, either *people or the planet*, and of Keynes, the dictum that "*practical men who believe themselves to be quite exempt from any intellectual influence, are usually the slaves of some defunct economist."* Datta's paper draws insights about levelling up from Islam and Christianity. Climate change underscores the interdependence of humans and their environment. Mantras about rewards being founded (albeit skewed) on merit and proportionate to the inequality in individual contributions to society and equality of opportunity were founded on the voices of some academic scribbler a few years back.

Datta's work on Zakat, the Islamic tithe, has several implications for the understanding of Islamic economics and the role of Zakat in Islamic society:

1. Zakat is a means of promoting economic growth and reducing inequality: Datta suggests that Zakat can stimulate economic growth by increasing the flow of money in the economy. By requiring Muslims to give a portion of their wealth to the poor, Zakat encourages the circulation of money, leading to increased economic activity and development.
2. Datta suggests that the state should manage the collection and distribution of Zakat rather than left to individual or private initiatives. This ensures that Zakat is collected and distributed fairly and efficiently and that the needs of the poor and vulnerable are met.

The Shapley value (SV) is a concept from cooperative game theory used to allocate the total payoff of a game among its players fairly and efficiently. It is based on the idea that each player's contribution to the game's total payoff should be measured by the marginal value they add when joining a coalition of players.

The interdependence perspective of Zakat suggests that the wealthy members of society have a responsibility to help the less fortunate and that their prosperity depends on the well-being of the community as a whole. The SV provides a framework for allocating the benefits of cooperation fairly and efficiently and can be used to illustrate how the contributions of each member of society are interconnected and interdependent.

The actual distribution of income and wealth is unevenly distributed, mainly in the UK and the US (table 1), increasing inequality. Inequality has a high social cost. Many factors contribute to this uneven distribution of income and wealth, including differences in education, skills, work experience, and systemic inequalities in access to opportunities and resources.

Table 1: UK & US Income and Wealth Distribution (2019-2022)

UK	USA
According to the Office for National Statistics, the top 10% of households in the UK held 45% of the country's total household wealth in 2020, while the bottom 50% held just 9%.	According to the Federal Reserve, the top 1% of households in the US held 15 times more wealth than the bottom 50% of households combined in 2020[i].
The Institute for Fiscal Studies reported that the top 1% of earners in the UK held 14% of the country's total income in 2019, while the bottom 50% held just 18% of the income[ii].	The top 1% of earners in the US held 20% of the country's total income in 2019, while the bottom 50% held just 12% of the income, according to the Economic Policy Institute[iii].
The Gini coefficient, a measure of income inequality, was 0.365 in the UK in 2022[iv].	The Gini coefficient in the US was 0.485 in 2022[v].

We contrast the principle of Zakat, which is based on spiritual principles with the secular and capitalist principle that the distribution of income and wealth in modern society should be based on what we see as arbitrary notions of merit on the one hand and entitlement on the other. We base our argument on Datta's economic analysis of Zakat and extend his analysis to take account of the random aspect of distribution implied by the SV and the fact that contributions to society result from group efforts rather than merely heroic individual efforts.

We set out a weighted Shapley value (WSV) concept as a theoretical underpinning for Zakat. WSV captures the elements of luck associated with distribution and the tendency of actual distributions to follow a power law. WSVs are an antidote to marginal productivity theories of distribution since they consider the interdependence of individual contributions to organisations. Under very general conditions, a weighted average of SV can be derived that fits any possible distribution of payoffs. Call these weights *power weights* which indicate the power of some groups in society and that their strength is determined by societal power. Bargaining power is in turn, determined by the formal and informal rules that govern that society.

WSV can be used to explain many distributions of personal income, including ideal distributions suggested by Zakat, the living wage, the current power law of distribution between rich and poor and Keynesian's proposition that the distribution of personal income will inevitably be skewed. However, the difference between rich and poor does not to be as great as it is. We argue for Keynes' proposition. The personal distribution of income is the weighted sum of choice, chance and coalition structure (Fellner, 1964; Sahota, 1978). Weights are arbitrary, the choice is restricted, and chance is a joker in the coalition structure. The choice is determined by heredity and environment. We are saying that heredity, environment and luck determine the current weights.

Personal Distribution of Income (PDI) = Choice x Chance x W x Shapley value

1. The personal distribution of income is a function of elements of choice, chance and WSV. The relevant literature is the (Friedman, 1953; Kaldor, 1955) SV, coalition structure, the economics of genes, and environment plus rent of ability/merit.
2. SV luck x contribution to a coalition

The rest of the paper is divided as follows. First, we examine Datta's paper, Zakat and SV implications. Second, coalitions and coalition structures are described. Third, Shapley's axioms

are set out and the extended SV is explained with an illustration (Pascal's Triangle). Fourth, we emphasise the link between Zakat and SV**,** and the final section discusses the implications and the conclusion.

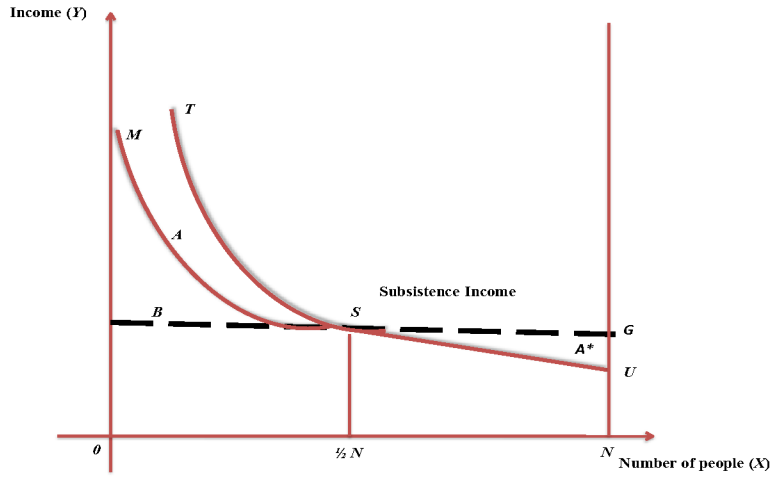
2. Literature Review

The research gap in the current literature in the application of distribution weights of Shapley value (DWSV) to Zakat distribution is the lack of empirical studies that have explored its effectiveness in achieving the objectives of Zakat. While DWSV has been widely used in other areas, such as cooperative game theory and resource allocation, its application in Zakat distribution is relatively new and requires further exploration. Therefore, this paper is an attempt to provide a refocus in understanding the potential of this method to improve the distribution of Zakat and ensure a fair and efficient allocation of the fund.

We extend Datta's approach by introducing distribution weights into the well-known SV. We set out the concept of weighted shapely values that retain the elements of randomness and marginal contribution to a coalition contained in pure/true shapely values and weights that follow a ley-Pareto distribution. We reproduce Datta's exposition in a diagram below, an adaptation of the original in his paper.

Figure 1 below shows the curve *TSU*, the density function as income distribution among the population *N*. The distance between *½N* and *N* means that exactly half of the population is below subsistence income. The incomes of the poor are assumed to be distributed linearly (indicated by the straight-line section *SU*). The other half is above the subsistence level (indicated by the curved line section *TS*). Areas *A* and *A** on the graph, i.e., area *TMS* = area *SGU*). Suppose we assume area *A* as Zakat for bringing the poor up to subsistence-level income. In that case, the remaining area *B* (indicated by *MBS*) is the excess income of the upper-income groups that can be used to accumulate capital and wealth in the future.

Figure 1: Datta's Diagram Income Distribution



In a sustainable community (a community that is not trapped in any famine or war) with stable or growing income or wealth, the sum of those incomes that exceed the agreed subsistence level must be greater or equal to the sum of those below it (Tamanni et al., 2019; Al-Banna et

al., 2019; Al-Bawwab, 2023; Hudaefi et al., 2022). Denoting the total number of people in a society as N (substituting N for K in inverse power law relation), figure 1 above depicts the following equation: $Y = ½NX^{-\alpha}$.

Here we focus on the first aspect as it closely relates Zakat with the Pareto distribution; wealth, income, poverty and Zakat. Distributive justice within Islamic society allows earnings differences as long as they obey the differences between the values of the service one contributes to society as a whole. Justice does not necessitate everyone to be regarded equally, irrespective of his contribution to society, but in an interdependent society, one's contribution is often dependent on the contribution of others. This is implied by the SV, which differs fundamentally from the marginal productivity viewpoint.

A Nobel Prize was awarded in 2012 to Lloyd Shapley, recognising his work in cooperative or coalitional games (Roth, 1988, 2016; Shapley & Shubik, 1954; Shapley, 1953; Tlemsani & Matthews, 2010; Hicks, 1939; Burk, 1938; Winter, 2008; Monderer et al., 1992). This paper maintains that one aspect of his work, the SV, has much greater theoretical and practical significance. The distinction is more artificial than is usually recognised. It is generally recognised that an enormous amount of injustice exists worldwide. The disparity of incomes within and between nations has increased.

Another feeling is that people are rewarded according to their individual productivity, irrespective of the contribution of a host of supporting and often menial efforts. One qualification to the marginal productivity argument is that element of choice, chance and windfalls can affect distribution Friedman (1953), but now remuneration is largely attributed to capability and performance rather than luck. In contrast, classical and pre-classical economists attributed income to ownership and monopoly power. Unlike neoclassical theory, they treated income distribution separately from the price mechanism Walsh and Gram (1980).

It is argued that the SV, as a solution value in cooperative games, is an antidote to this discourse. Although it is usually hedged by fairly restrictive assumptions, these assumptions can be conveniently and realistically relaxed so that the scope of the theory can be extended whilst preserving its essentials. The SV implies that there are many interdependent contributions by multiple stakeholders to value, which consists of various payoffs, positive and negative. The SV is not dependent on the existence of equilibrium. It can be extended to explain actual distributions of income and wealth. It is more in keeping with the complex global systems that exist now than the prevailing discourse and it accords with aspects of fairness, to which there is widespread consent.

Cooperative games are about how groups or coalitions create payoffs or side payments by cooperation in coalitions in situations where in contrast to non-cooperative games, agreements formal and informal such as contracts, promises or threats, are binding and enforceable (Brams, 1975; Grief, 2008; Luce & Raiffa, 1957). Shapley showed that the SV exists under certain conditions as a unique payoff vector to the players in the game. This is the expected marginal contribution to a random coalition C. The coalition is a very broad term. In business and economics, coalitions are made up of hierarchies of sub-coalitions, teams, projects, value chains, businesses, functions, and corporations and given the relationships within which they exist, it is difficult to draw boundaries between them.

Given assumptions of symmetry, linearity, and exclusion of players or activities that do not contribute to payoffs, Shapley demonstrated that a unique payoff value, the SV of each player

in a coalition, exists and that the sum of these SV is exactly equal to the payoff value of the coalition. It is the weighted average of the expected marginal payoff contributions to the grand coalition of all activities and all the sub-coalitions, it is composed of the sum of the SV, if there are K players, is exactly equal to the payoff value of the grand coalition consisting of all K players. Expected payoffs are calculated by; (a) using weights made up of permutations of coalitions that agents in the coalition can be marginal to and expressing each one as a fraction of total permutations, [K!].

Coalitions are assumed to occur randomly. Unanimity is assumed to be a condition for entering or sub-coalition. Unanimity implies that coalitions are Pareto optimal. This is akin to the exhaustion of the product theorem, based on the exhaustion of the product (Euler's) theorem of neoclassical theory. Of particular significance are the implications of the SV. The authors proposed the following implications.

Implications

1. Individual members of a coalition make multiple contributions that are often unrewarded.
2. The contribution of one member of a coalition is dependent upon the contribution of others.
3. The measure of contributions is payoffs, which have both monetary and non-monetary aspects; transferable payoffs or utilities are usually assumed.
4. The significant agents in an organisation are stakeholders rather than the usual categories, managers, staff, shareholders and so on.

On the grounds of these implications, the SV provides a benchmark for a fair distribution of payoffs. The interpretation of payoffs in both monetary and non-monetary terms relates rewards more closely to welfare than usual definitions of income. We call the SV the true SV. The existence of the true Shapley value is contingent on some fairly restrictive axioms, the most restrictive being that it holds only when the grand coalition offers maximum payoffs. This is a unanimity or Pareto optimal situation associated with competitive equilibrium and the core of a competitive economy.

A fifth implication forms the main subject matter of the paper.

5. Under very general conditions, a weighted average of SV can be derived to fit any possible payoff distribution. Call these weights the *power weights*. Power weights indicate the power of some groups in society and the strength of power weights is determined by bargaining power in a society. Bargaining power is, in turn, determined by the formal and informal rules (grammar [G]) that govern society.

For our purposes, we call the 5 implications collectively the significance of the extended Shapley value (ESV) in contrast to the true Shapley value. The ESV breaks with the Shapley axioms but admits the SV under Shapley axioms as a possible distribution. This means assuming that the set of all possible distributions is a convex set or forming a convex hull of all possible distributions, assuming that this device accords with feasible and attainable distributions and including the SV as a feasible and attainable distribution. Possibility, as used here includes feasibility and attainability. Feasibility means that the sum of payoffs to stakeholders does not exceed the total payoffs available. Attainability means that the formal and informal conventions (the grammar [G] of society) are such as to permit it to happen.

Given implications 1-5 above an argument can be made that SV are fair in that they take account of the entire contributions of an individual agent or entity. The ratio of the actual payoff to the Shapley indicates the individual's power in allocating income to participants in a coalition or coalition structure. The reasons for social inequality are complex, but the growth of monopoly and oligopoly power in the global economy, a winner takes all culture among some sections of the community and the decline of trade unions is no doubt partly responsible for the influence of the power ratio of different members of society.

In the Zakat context (Tamanni et al., 2019; Al-Banna et al., 2019; Tlemsani & Matthews, 2020; Al-Bawwab, 2023; Hudaefi et al., 2022; Bin-Nashwan et al., 2020, 2021) examine the socio-economic merits of Zakat and its potential role in eradicating poverty. They argue that Zakat can help to build a sense of community and reduce social and economic inequality. The study found that Zakat serves as a means of social justice and solidarity, promoting the well-being of the individual and society. Additionally, Zakat can stimulate economic growth by providing capital to those who lack resources and promoting entrepreneurship.

One approach to distributing Zakat that has gained attention in recent years is the use of the weighted Shapley value, a concept from cooperative game theory that allows for a fair distribution of resources among a group of players. It is based on the SV, which represents the average contribution of each player in a game to all possible coalitions, taking into account the order in which the players join the coalition.

In the context of Zakat, a distribution based on the WSV would mean that the amount of Zakat received by each beneficiary would be proportional to their needs and their contribution to society. This could involve assessing the needs of each recipient and assigning them a weight based on their level of need. For example, a person who is disabled and unable to work would receive a larger share of Zakat than a person who is non-disabled and has a stable source of income.

Abu-Tayeh et al. (2019) proposed using the WSV to distribute Zakat funds among the poor and needy in Jordan. The researchers used data on the income and expenses of households in Jordan to calculate each household's "contribution score" based on their income and the number of dependents they support. They then used the WSV to allocate Zakat funds among households, taking.

3. Coalitions and Coalition Structures

In business and economics, organisations are made up of coalitions of activities within activities, teams, projects, functions, value chains, divisions, businesses, and corporations. Given the prevalence of supply chains, alliances, partnerships and mergers, it isn't easy to define the boundaries of organisations. Activities include producing, consuming, storing, financing, consuming, organising or disposing of something. Organisations are made up of (sub) coalitions within coalitions and hierarchies of activities. In any coalition, the grand coalition consists of all activities. This has a special place in welfare economics; in competitive equilibrium, the grand coalitions are Pareto optimal. In competitive analysis, the grand coalition contains all its subset coalitions. Unanimity means that coalitions are made up of a hierarchy of PO sub-coalitions- given the significance for SV understated.

Coalitions are founded on fundamental activities [A]. The definition of fundamental activities is as elusive as that of fundamental particles. They are thought of here as being something that

every higher-level activity is made of. The organisation consists of partitioning fundamental activities into successive groups or coalitions, a mapping [G] of fundamental activities into a matrix of payoffs [B].

A convenient way of thinking of fundamental activities is as pure information, which is transformed into a message of some kind; the nature of the message is determined by the grammar [G] used to interpret it. It is sufficient here to outline our conception of grammar as a complex form of conditioning that has formal and informal aspects. It includes rules, laws, regulations, routines, cultures, memes, conscious and unconscious determiners of our ways of thinking and acting. Grammars range from the public to the individual scale. Here we are concerned with the grammar that influences income distribution and the alternative grammar that gave rise to Zakat and meritocracy.

Coalitions and organisations, which are made up of coalitions within coalitions map activities into payoffs and in turn distribute payoffs according to their particular grammar at a point in time.

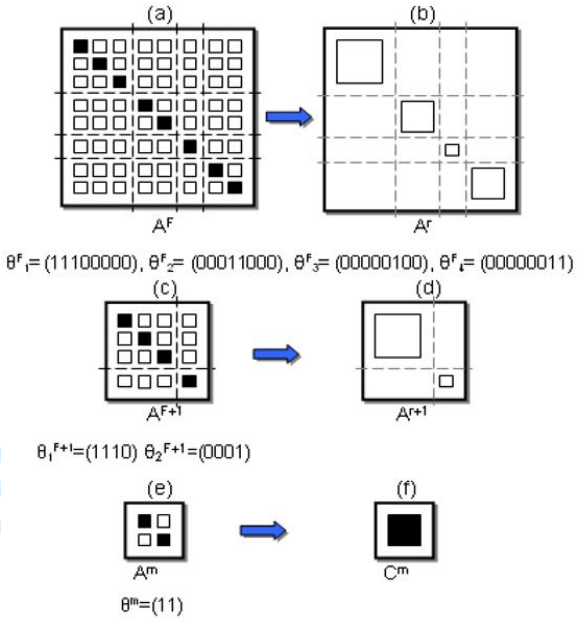
In this paper, we denote coalitions as C, the typical coalition with K members is C^K, and the value of C^K is called B^K. The payoff value of the grand coalition of all K activities is called $H[B^K]$. An individual member of C^K is called s, and the set of all such individual members is called S.

Coalition structures are denoted as L, which in the case of businesses, is a partition of all the N fundamental activities that exist into a set of disjoint coalitions; $L = \{C^1, C^2, ..., C^K, ... C^N\}$. When we are speaking about coalition structures in terms of payoffs L^P, in which case $L^P = \{B^1, B^2, ..., B^K, ... B^N\}$. The existence of coalitions or coalition structures is not dependent on the existence of equilibrium. Organisations or any system always exist in some coalition structure or other.

If Figure 2 below is seen as a process of forming coalitions at successively higher levels in the hierarchy, fundamental activities A^F in Figure 2(a) are combined into 4 coalitions (C^1, C^2, C^3, C^4) corresponding to A^r in Figure 2(b). New higher-level complementarities may appear, A^{F+1}, in 2(c), and be combined into two possible coalitions A^{r+1}, made up of $\theta_1^{F+1} = (1110)$ and $\theta_2^{F+1} = (0001)$. These offer higher level synergies in the organisation matrix A^m when a grand coalition (C^m) is formed.

From the fundamental activities A^F, a series of coalition structures are formed, L^P_F, L^P_{F+1}, L^P_{F+2}, L^P_{F+m} in figure 2(a) to 2(f); the binary strings θ^F, θ^{F+1}, θ^m illustrate the choice.

Figure 2, Process of Forming Coalitions



Coalitions themselves can be considered a hierarchy of coalition structures; coalitions within coalitions generally contain increasing numbers of fundamental activities as we proceed up the hierarchical coalition structure.

4. The True Shapley Value

The extensive developments in game theory over the past 70 years owe much to the work of (Neumann & Morgenstern, 1947; Roth, 1977; Shapley & Shubik, 1954; Nash, 1951; Tlemsani & Matthews, 2021). The central idea of the SV is that society is made up of teams or coalitions that create (or destroy) value. Any individual is a member of many coalitions or teams, such as families or organisations in which he/she works voluntarily or in paid employment, neighbourhoods or communities in which he/she lives, mosques, churches or charities that extend to entire societies and so on. The term coalition describes these diverse groups, and the setup is as follows: a coalition of players cooperates and obtains a particular overall gain from that cooperation since some players may contribute more to the coalition than others or may possess different bargaining power (for example, intimidating to abolish the entire surplus). What final distribution of generated surplus among the players should arise in any game? Or how important is each player to the overall cooperation and what payoff can he or she reasonably expect? The SV provides one possible answer to this question.

It is helpful to give a preliminary account of the SV concerning Zakat. The SV is a solution concept in cooperative (or coalitional) games. Games are cooperative if commitments, including formal and informal rules, traditions and agreements, are binding. Pioneers of cooperative games were (Neumann & Morgenstern, 1944). The solution concept means that if all members (or players) of a society (or game) receive payoffs according to their Shapley, this equals the total payoffs. The sum of SV equals the game's value (total payoffs). As far as we know, this paper is the first application of their concepts to Zakat.

The SV has two important aspects, one explicit and the other implicit. The explicit aspect is that the SV performs the same function as Euler's theorem does for marginal productivity theory. Under the assumption of constant returns to scale, awarding factors of production according to their marginal productivity exhausts national income. The SV takes an entirely

different tack. Instead of factors of production, the SV is couched in terms of coalitions and coalition structures. A coalition structure may consist of an array of coalitions. Alternatively, it may consist of a single or grand coalition of every agent in the economy. In both cases division of rewards according to SV exhausts national income, the difference being that the grand coalition of all agents is a Pareto optimal distribution. The same result as that of perfect competition.

From the author's perspective, the most important properties of the SV are its implicit properties. Firstly, rewards are distributed randomly in a coalition. Secondly, rewards coincide with an agent's marginal contribution to a coalition. Therefore, the SV implicitly abandons the notions of ability and merit, and distribution is random according to the SV. An agent's standalone contribution may be very different from the contribution to the coalition as a whole. In other words, contributions are interdependent.

SV is a solution concept in cooperative (or coalitional) games. The solution concept means that if all members (or players) of a society (or game) receive payoffs according to their Shapley, this equals the total payoffs. The sum of SV equals the game's value (total payoffs). According to Shapley, the contribution that an individual generally makes to society is the (weighted) sum of the contributions (the differences) he/she makes to all the coalitions he/she is a member of. Shapely showed that all individual contributions, suitably weighted, added up to the total value created by society.

The easiest way to see the activity of true SV is to consider a coalition consisting of K members and expand Pascal's triangle (Figure 3) to the K^{th} row. There are then 2^K ($0 \le K$) possible coalitions, including the empty coalition. If coalitions occur randomly, each has a probability of $1/2^K$. Now, consider the $(K\text{-}1)^{th}$ row This reveals the $[2^{K-1} - 1]$ activities that activity s can be marginal to. If these are divided by a permutation of K ($K!$) to provide weights $S!(K-S-1)$ on marginal contributions, $\left[B^K \cup s - B^K\right]$, then double counting of marginal values is avoided. The Shapley value of an activity φ_s is:

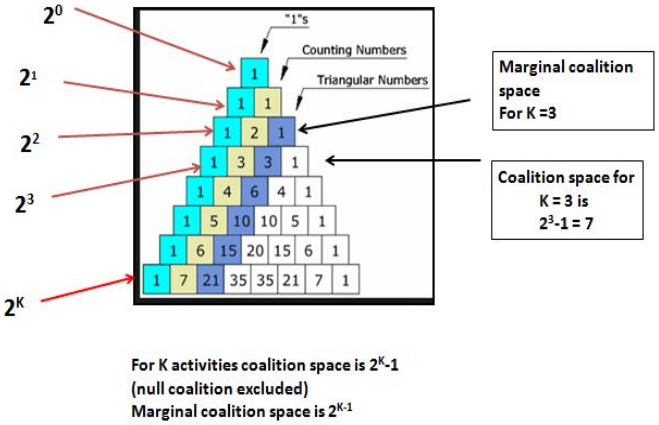
$$\varphi_s = \sum_{S \subseteq C^K - s} \frac{S!(K-S-1)!}{K!}\left[B^K \cup s - B^K\right] \qquad (1)$$

To reiterate, the first terms on the right-hand side of equation (1) are probability weights, made up of permutations of sub-coalitions of (s) *activities* in B^k, and the value of coalition C^K has K activities. The second term comprises increments in the payoffs of activities (s) to all sub-coalitions and marginal contributions. Their r sum, $\sum_{all \sin C^K} s$, i.e., is a unique value, the payoff value of the grand coalition $H[B^K]$ of the K activities in C^K that satisfies Shapley's axioms.

$$H[B^K] = \sum_{all \sin C^K} \varphi_s \qquad (2)$$

In a coalition of 3 activities, for example, $H[B^K] = \varphi_1 + \varphi_2 + \varphi_3$

Figure 3, Pascal's Triangle



For example, consider 3 activities *a*, *b*, and *c* (K = 3). There are 2^3–1 = 7, possible sub coalitions, excluding the null coalition {0}; {abc},{abd},{bcd},{ab},{ac}{bc},{a},{b},{c}. There are $2^2 = 4$ sub-coalitions that any activity can be marginal to. For example, activity a can be marginal to one two-member {(a) b, c}; two one-member {(a) b} and {(a) c} (table 2). The SV is formed by arranging a permutation of these sub coalitions, according to the number of activities in the queue and expressing them as a proportion of the number of possible permutations of 3 activities 3! one two-member 2!0!/3!; two one member, 1!1!/3!; one empty sub coalition 0!2!/3!.

These are used as weights, 1/3, 1/6, and 1/3, respectively. Suppose the marginal values are 19 to {bc}, 9 to {b} and 11 to {c} and 1 to {0}. The SV of a, φ_a is 19/3+9/6+11/6+1/3=10.

Table 2, Pascal's Triangle

Aggregate Coalitions $K = 0, 1, 2$ & 3				Marginal Coalitions $K = 0, 1$ & 2 $s \subseteq S$ & $S \subseteq K$			Weight
[1] Members of coalition	[2] Size of coalition C^K	[3] Number of coalitions	[4] Value B^K	[5] Members (.) is the marginal member	[6] Number of marginal coalitions	[7] Value $(B^K \cup s)$-B^K	[8] $\frac{S!(K-S-1)!}{K!}$
{abc}	3	1	45	{(a)bc} {(b)ac} {(c)ab}	3	19 25 31	$1/3 = \frac{2!0!}{3!}$
{ab} {ac} {bc}	2	3	14 20 26	{(a)b} {(a)c} {(b)a} {(b)c} {(c)a} {(c)b}	6	9 11 13 17 21 19	$1/6 = \frac{1!1!}{3!}$
{a} {b} {c}	1	3	1 5 9	{(a)0} {(b)0} {(c}0}	3	1 5 9	$1/3 = \frac{0!2!}{3!}$
{0}	Dummy						

In a coalition of 3 activities, for example, $H[B^K] = \varphi_1+\varphi_2+\varphi_3 = 45$, is illustrated in table 3 below. SV is calculated in the same way as in the paragraph above.

Table 3, Shapley value calculation

$$\varphi_a = \frac{2!0!}{3!}19 + \frac{1!1!}{3!}[9+11] + \frac{0!2!}{3!}1 = 10$$

$$\varphi_b = \frac{2!0!}{3!}25 + \frac{1!1!}{3!}[13+17] + \frac{0!2!}{3!}5 = 15$$

$$\varphi_c = \frac{2!0!}{3!}31 + \frac{1!1!}{3!}[21+19] + \frac{0!2!}{3!}9 = 20$$

The SV is the unique value that satisfies the Shapley axioms of *symmetry*, *null (dummy) player*, *efficiency,* and *additivity*. *Symmetry* means that agents who make the same contribution to a coalition receive equal payoffs. *Additivity* means that the value of any two coalitions or sub-coalitions together equals their value separately. If an agent adds nothing to a coalition, according to the *null or dummy player* axiom, he or she receives no payoffs. The axiom of *efficiency* and feasibility means that the sum of the payoffs to all the agents in the coalition is equal to the value of the grand coalition, as in table 2 above. That the value of the grand coalition is efficient in a Pareto sense. Pareto optimality[1]It is only immediately apparent if an implicit property unanimity is made explicit. Unanimity means that if agents have greater payoffs if they choose coalition *x* rather than coalition *y* then *x* will be chosen. Hence the coalition they choose must have the highest payoff...fs. So, the grand coalition of *K* players must yield the highest payoffs.

$$\sum_{s=a,b,c\subseteq C^3} \varphi_s = H[B^3]$$

Notice in the table that satisfies Shapley's axioms, in that.

i. If the empty coalition is excluded, the values lie in (2^3–1) vector space.
ii. Every player makes a positive contribution and *null (dummy) players* who make zero contributions are excluded.
iii. Marginal contributions to sub-coalitions are unchanged when higher level coalitions are formed, *symmetry,*
iv. SV is added, *additivity,*
v. Their sum is the value of the grand coalition $H(B^3)$, *efficiency (Pareto optimality, unanimity)* but unanimity.

The SV is the unique value that fits the Shapley axioms. Intuitively this can be seen in Tables 2 & 3 above. The uniqueness of the SV follows from the 2^{K-1} dimensions of coalition space when there are *K* agents or stakeholders. Shapley's axioms exclude the empty coalition. The basis vectors for a space of 2^{K-1} dimensions are provided by 2^{K-1} linearly independent coalition vectors. Consider these coalition vectors as a set of efficient (*unanimity or Pareto optimal*) games. Consider an efficient value of each game in the set as 1 (any efficient value, for example, $H[B^K]$, will do). There are *K* agents in the game they will share this *efficient* value equally, each getting a payoff of *1/K* since there is only one solution to the problem and we have identified the SV as an *efficient* solution (for every player and everyone of the 2^{K-1} coalitions in the space), then the SV is unique. Call this the true SV.

4.1 Extended Shapley value

If the SV is a unique solution to the problem set by the Shapley axioms, we can only arrive at an alternative value if we change the axioms. One candidate is the exclusion of the agent who

[1] Suppose the coalition is super-additive, meaning that every activity and every sub-coalition adds to the value of the coalition as a whole. In that case, the grand coalition of all *K* agents is Pareto optimal. Should a grand coalition not be Pareto optimal, the unanimity rule states that it will be rejected for a better one, a grand coalition that is in fact, Pareto optimal.

makes a zero or negative contribution. Another is the efficiency axiom. Inefficient outcomes occur. Most likely, they are the dominant outcomes. Suppose the set of all possible payoffs [D] is a convex set. This means that for any two distributions [D_I] and [D_J] and $\lambda[D_I]+(1-\lambda)[D_J]$ for $0 \le \lambda \le 1$ and $\sum \lambda = 1$. This means that any two distributions are included in the set. Also, include SV in the set, this means forming a convex hull of all possible distributions and including Shapley distributions. A convex hull of a set of points is defined by the smallest set that contains those points.

Let [D_I] be a distribution according to Shapley values and [D_J] be any other feasible distribution. Denote the power ratio, the ratio of the actual payoff of activity s, f_s, to its Shapley payoff φ_s and their ratio as ρ_s. Then

$$\rho_s = \frac{f_s}{\varphi_s} \tag{3}$$

for $$0 \le \rho_s \le \frac{B^K}{f_s} \text{ and } \sum_{all\,s\,in\,C^K} f_s \le B^K \tag{4}$$

Let us call ρ_s the bargaining power of s remembering that we have identified activity s with stakeholders. The extended SV is more general than the actual SV. As equation (4) shows, it applies to the oligopolistic and monopolistic trends in the global economy to a winner takes all philosophy, decline in union power and less progressive taxation. However, it takes into account all implications (1-5) above. As the second set of inequalities in (4) shows, it applies to non-inefficient as well as efficient situations and it does not exclude dummy or negative players.

4.1.1 Illustration of extended SV

As an illustration, consider a payoff matrix with three activities with Shapley payoffs, as in Table 3 above:

$$\varphi_1 = 10, \varphi_2 = 15 \; and \; \varphi_3 = 20 \quad \text{and} \quad H[B^K] = 45 \tag{a}$$

Consider any feasible distribution of payoffs, for example,

$$f_1 = 30, f_2 = 14, f_3 = 1 \quad \text{and } B^K = 45 \text{ a Pareto optimal solution} \tag{b}$$

$$\text{Or} \quad f_1 = 25, f_2 = 10, f_3 = 1 \quad \text{and } B^K = 36 \tag{c}$$

Clearly, (c) is a sub-optimal solution. But both are feasible. The difference is that (b) is high ρ_2 and is relatively weak in comparison to ρ_3 which is exceptionally weak in both (b) and (c).

5. Shapely Value and Zakat

Zakat relates to the SV, an essential concept in the theory of cooperative games, seeing Zakat as contributing to social capital. Payoffs are a central concept in game theory and they include (a) monetary and non-monetary elements (b) payoffs that accrue to others in society as well as payoffs that accrue solely to the individuals who create them and (c) both spiritual and material values. This conception of payoffs is fitted closely with that of Zakat, which sees the spiritual

and the material as interdependent worlds. Our conception of payoffs is closer to that of Zakat, it differs from conventional economics, which sees factor incomes (wages, salaries, rent and profit) and distribution through the lens of marginal productivity theory and sharply distinguishes spiritual and material worlds.

The SV illustrates the interdependence perspective of Zakat. Payoffs apply equally to both givers and receivers. They are formed by individuals who willingly cooperate in society rather than atomistic individuals who act independently. People cooperate in groups or societies to create value. The SV illustrates that (c) gains from cooperation are the rationale for living in a society and (d) one person's contribution depends on another's contribution.

The idea behind Zakat is not just to provide financial assistance to the needy but also to promote social justice and strengthen the bonds between members of the community. In this sense, Zakat can be seen as a recognition that society needs to be cooperative rather than individualistic, that society is a collective entity and that individuals are responsible for the well-being of the community as a whole. It is a manifestation of the Islamic principle of social justice and a means of promoting solidarity and cooperation within society.

Zakat aims to reduce socio-economic differences by distributing wealth and income. The question raised here is how Zakat, as a religious obligation, relates to the power-law distribution and its relation to the Pareto distribution. There are two aspects to the question. The first relates to distribution and the second relates to the broad issue of how power laws seem to express surprising regularities in nature. Here the subject matter is economics and income distribution (the association of power laws with extreme events like social revolutions and the transformation of entire societies, for the better or worse, as the current upheavals in the Middle East). Here we focus on the first aspect as it most closely relates Zakat with the Pareto distribution, wealth, income, poverty and Zakat.

Social inequality is complex, but with the growth of monopoly and oligopoly power in the global economy, a winner takes all culture among some community sections. The decline of trade unions is undoubtedly partly responsible for the influence of the power ratio. Inequality is often justified by *a priori's* of the predominant business model the notion that individuals can and should seek competitive advantage, that this encourages innovation, the growth of GDP, society's welfare, and abnormal rewards prevent the individuals who receive them from society from leaving the country. Another feeling is that people are rewarded according to their contribution or productivity. The qualification to the productivity argument is that element of choice and chance, windfalls can affect distribution.

Why the Shapley's value, power laws, and coalition structures need to be contextualised in the Zakat administration? Shapley's values, power laws, and coalition structures are mathematical concepts commonly used in cooperative game theory and social choice theory to understand the distribution of payoffs among players in a cooperative game and to analyse the distribution of benefits among members of a group or coalition.

In the context of the Zakat administration, Shapley's values can be used to determine the fair distribution of Zakat funds among eligible recipients. These values provide a way to allocate resources fairly based on each group member's contributions. In this case, the group is composed of eligible recipients of Zakat funds. For example, while Shapley's values may provide a fair way to distribute funds among eligible recipients, Islamic teachings emphasise

the importance of prioritising those in need and those who are closer to the donor in terms of family or community ties.
Power laws, on the other hand, are a mathematical concept that describes the distribution of resources within a system. In the context of the Zakat administration, power laws can help us understand the distribution of wealth among eligible recipients and identify potential areas of inequality that need to be addressed.

Coalition structures refer to the formation of subgroups within a larger group. In the context of the Zakat administration, this could refer to the formation of smaller groups of eligible recipients based on their needs or specific circumstances. By understanding the potential coalition structures that may form, Zakat administrators can ensure that the distribution of Zakat funds is equitable and that all eligible recipients receive the assistance they require.

6. Further Discussion

The novelty of this paper is that it blends the SV and the idea behind Zakat by introducing the idea of alternatives of shapely weights. The philosophy behind Zakat is that national wealth should be shared and income distributed more fairly than it is. Evidence (data) shows that this inequality has a high social cost.

Many societies worldwide claim to be meritocracies, that their societies exclusively distribute resources based on merit. However, many meritocracies have been discredited in a paradoxical way politicians talk about evening up. There is a reluctance to make taxes more progressive, which is the reason disincentive. We got an ambivalent attitude. Inequality of income and wealth in modern societies is so commonplace (Alvarado, 2011; Tlemsani, 2010; Atkinson, 2005). For example, in 2018, the richest 20% of UK households received 47% of all income, whereas the most deficient 20% earned only 4%. The gap is even more significant in terms of wealth inequality, where 10% of the UK households possess 44% of all wealth and the most deficient 50% own just 9%. This level of disparity is not unavoidable with the current practices: today, inequality/wealth is considerably more significant than it was in the 1950s and 1960s…. Etc. and will be smaller than in the 2030s and 2040s…Etc. In 2019, total global wealth rose by $36.3 trillion, up 8.5% versus 2018, according to Credit Suisse Global Wealth Report 2020 (Tables 4 & 5).

Table 4, Global Wealth Inequality (The Richest 1% Own 44% of the World's Wealth 2020)

Wealth Range	% of Adults	Wealth Share %
$1,000,000 +	0.009	0.439
$100,000 - $1,000,000	0.098	0.389
$10,000 - $100,000	0.326	0.155
under $10,000	0.566	0.018

Table 5: Global Billionaires Make Trillions as Workers Lose Trillions

F1	Total in $
Billionaire Wealth Gained (18/03/20 – 31/12/20)	$3.9 tn
Labour Income Lost (2020)	$-3.7 tn

The extended power ratio is quite general and retains a little of the elegance of the true SV. The extended value is justified because it is general and may provide an antidote to current preconceptions about the justice of remuneration. Another aspect of the generality of the extended value is that it applies not only to equilibrium but to disequilibrium situations. Whatever the system state, coalitions exist, and the macroeconomy is in some coalition structure or other and likely since a modern economy contains so many interacting variables, households, firms and products, it lies in an attractor bounded by [*G*] than point equilibrium.

7. Conclusion

We chose the SV perspective because not only it illustrates the independence of one person's contribution upon that of others (in different occupations, different states and from different social classes), but it also illustrates the observation that justice requires that we recognise interdependent contributions explicitly by addressing questions of inequality, injustice and poverty. When norms and resources are allocated unequally among peoples and nations, social inequality arises due to the unfair distribution of wealth and the lack of access to opportunity.

Inequality is the root of many problems in modern societies. The huge gaps between rich and poor are excessive and unjustified. This leads to a sense of grievance and resentment that threaten democracy. The SV and Zakat provide valuable insights into the problems of inequality, especially in countries at the top of inequality tables, like the US and the UK.
The case for considering institutionalised almsgiving as a form of social capital is inescapable. The social capital aspect of Zakat is also tied with our previous perspectives; giving redistributes wealth and income and doing so dilutes inequality, injustice and poverty, which partly are sources of extremism. Further, the increase of social capital depends on investment in cooperation and reward systems, implied by the Shapley interpretation of contributions to society.

One potential area for future research is to investigate how DWSV can be used to improve the efficiency and effectiveness of Zakat distribution in achieving its poverty alleviation objectives. This could involve designing and implementing experiments or simulations to evaluate the impact of DWSV on the distribution of Zakat funds and the resulting reduction in poverty. Future research could also explore the potential of applying the WSV method to the Zakat distribution system. The research could focus on identifying the critical factors that influence Zakat's distribution and evaluating the method's efficiency and fairness compared to the current distribution system. Additionally, the research could investigate the perceptions and attitudes of Zakat recipients and donors towards applying the WSV method in Zakat distribution.

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